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SHORTER COMMUNICATIONS

DIFFUSIVITY MEASUREMENT METHOD BASED ON THE CONCEPT OF GROUP VELOCITY

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NOMENCLATURE

D, diffusivity;
t, time;
x, position;
ϕ, temperature.

INTRODUCTION

THE DETERMINATION of the thermal diffusivity D is usually based on one of the solutions of the heat conduction equation

$$D\nabla^2\phi = \partial\phi/\partial t, \qquad (1)$$

where ϕ is the temperature. The methods are divided by Carslaw and Jaeger ([1] pp. 25–26) into steady-state, periodic and variable state methods, depending on the class of solutions of (1) chosen.

With the advent of microprocessor electronics techniques the flexibility of programming and processing for special purpose systems is virtually unlimited. The choice of the physical configuration and the solution of (1) to be employed is therefore much larger presently.

The method suggested here uses a solution of (1) which closely resembles the wave packet concept, familiar from wave propagation theory. The method is characterized by a solution which starts with zero (arbitrary reference) temperature, is continuous and smooth, and the diffusivity can be determined from the propagation of the temperature null down the tested specimen.

THEORY

The exponential "wave"

$$\phi = \phi_0 \exp(-kx+\omega t), \qquad (2)$$

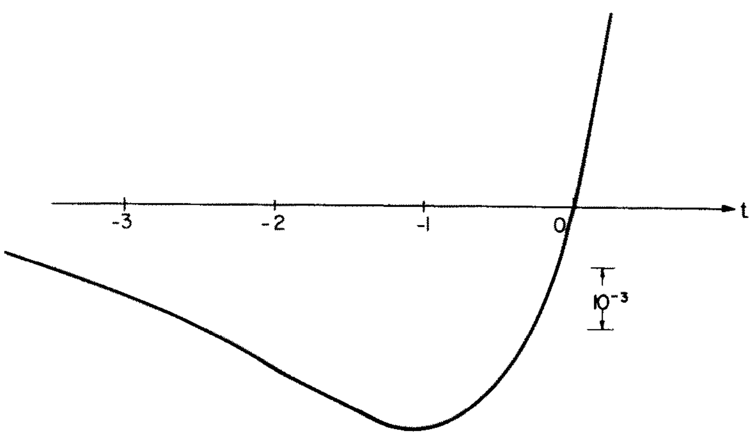

FIG. 1.

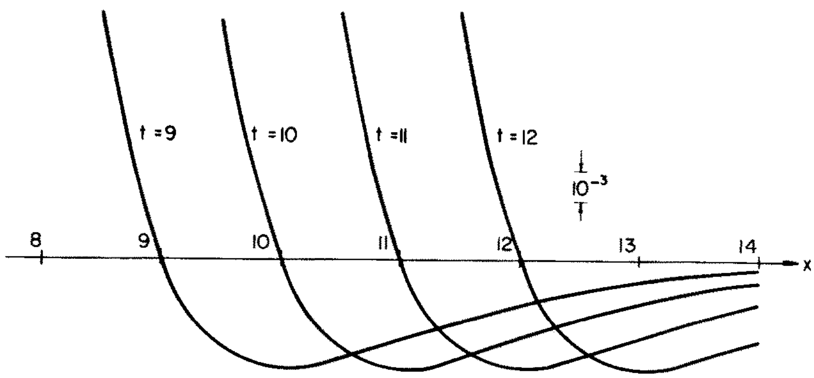

FIG. 2.

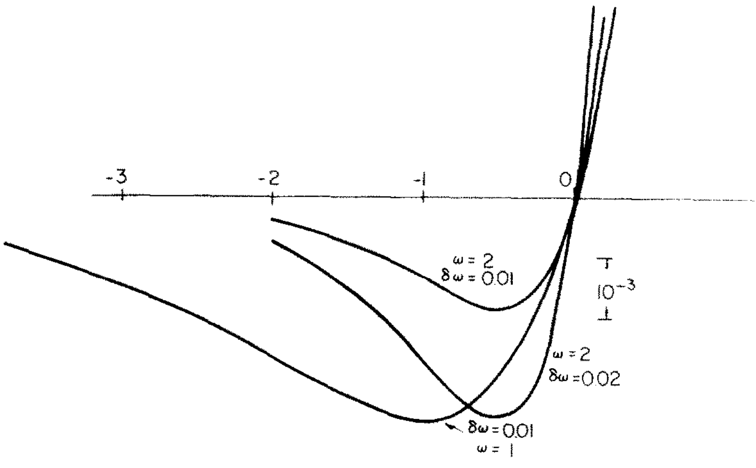

FIG. 3

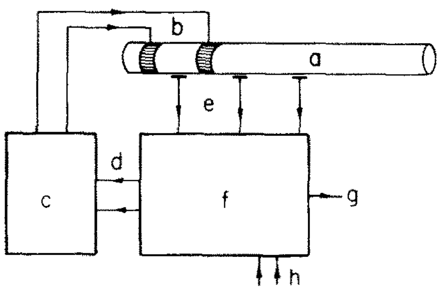

FIG. 4.

with ϕ_0 = constant, satisfies (1), provided k, ω are related according to the "dispersion equation"

$$F = \omega - Dk^2 = 0. \tag{3}$$

Motivated by the concept of the wave packet in wave propagation theory [2], we chose a combination of two waves as in (2) with slightly different k, ω,

$$\begin{aligned}\phi &= \tfrac{1}{2}\phi_0\{\exp[-(k+\delta k)x+(\omega+\delta\omega)t] \\ &\quad -\exp[-(k-\delta k)x+(\omega-\delta\omega)t]\} \\ &= \phi_0\exp(-kx+\omega t)\sin h(-\delta kx+\delta\omega t).\end{aligned} \tag{4}$$

Formally we have constructed a "wave packet" where the exponential is the "carrier wave", while the hyperbolic function is the analog of the "envelope". This is a solution of (1) provided (3) is satisfied, or for $\delta\omega$ δk small enough to be treated as differentials, we have

$$\frac{\delta\omega}{\delta k} = 2Dk. \tag{5}$$

In analogy to wave theory (5) is recognized as the group velocity [2]. It is clear however that this concept has only a limited application here. Only if we wish to follow $\phi = 0$, the reference temperature, then the group velocity is easy to track.

The function (4) for $x = 0$ is depicted in Fig. 1. The choice of parameters is arbitrary, $\omega = 1$, $\delta\omega = 0.01$. The behaviour of this function is such that it starts asymptotically from zero, goes through a minimum and then grows exponentially. In order to see how the point of zero temperature travels along the specimen, (4) is depicted (Fig. 2) for various times. Again $\omega = k = 2D = 1$, $\delta k = \delta\omega = 0.01$ are chosen arbitrarily.

For $\delta k \ll k$, the gradient of (4), which is proportional to the heat flux, has the same structure as (4). This means that the group velocity for the zero temperature point is actually the velocity of the heat energy. There are other velocities associated with heat conduction (or diffusion) systems. From (3) it is clear that the analog of the phase velocity, which here has but little relevance, is

$$\omega/k = Dk. \tag{6}$$

Hence the group velocity (5) is twice the phase velocity. From this point of view the heat conduction system exhibits what in wave theory is called normal dispersion. The velocity of the temperature nulls in a time-harmonic driven system ([1] pp. 64–68) is given by yet another expression

$$\sqrt{2Dk} \tag{7}$$

which is the geometrical mean of (5) and (6).

The shape of the input signal, Fig. 1, can be controlled by judiciously selecting ϕ_0, ω and $\delta\omega$. While ω governs mainly the width of the negative part, for fixed $\delta\omega$, the change of $\delta\omega$ while ω is held fixed affects the amplitude of the negative part. See Fig. 3, which is drawn for identical ϕ_0.

DESIGN CONSIDERATIONS

The specimen can be taken as a thin, thermally insulated rod (we have here in mind good thermal conductors, e.g. metals). An electrical heater is controlled by the microprocessor-controller. The heater affects the specimen at one end, but for reasons given below, supplies heat also to regions away from the end. Adequate temperature transducers feed information to the processor which controls the heater and computes D as well. The latter function is achieved by monitoring the motion of the temperature null as it moves down the specimen. This is schematically shown in Fig. 4. The tested specimen a is put in an insulating bore and heated electrically at points b, by a power supply c which is controlled by the processor-controller through d. The temperature is monitored at various points e, which feed into the microprocessor f. At its output g we get D, and the shape of the input temperature function (Fig. 1) is fed through input h.

The weak point of the method seems to be the high temperature needed to drive the null down the specimen. Above the crossover zero temperature the curve becomes exponential and the risk of encountering nonlinear processes is clear. However, once the temperature null has passed a certain point, more heating elements can be activated to further drive the process. From Fig. 2 it is clear that the slope in the exponential regions is almost the same for different times, hence no great complications are expected in implementing this idea. This and other points might be further evaluated once a working prototype is constructed.

REFERENCES

1. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, Second edn, pp. 25–26, 64–68. Oxford University Press, Oxford (1959).
2. L. Brillouin, *Wave Propagation and Group Velocity*. Academic Press, New York (1960).